

# The Luminosity-Temperature Relation for Groups and Clusters of Galaxies

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## ABSTRACT

We model the effects of shocks on the diffuse, X-ray emitting baryons in clusters of galaxies. Shocks separate the infalling from the inner gas nearly at equilibrium, and dominate the compression and the density gradients of the latter in the dark-matter potential of the cluster. We find that, independently of the detailed shape of the potential, the density gradient is steeper and the compression factor larger for the richer clusters. We show, considering the different merging histories, that in the hierarchical cosmogony the above effects lead, in X-rays, to a luminosity-temperature relation  $L \propto T^5$  at the scale of groups which flattens down to  $L \propto T^3$  for rich clusters in accord with the observations, and then saturates toward  $L \propto T^2$  for higher temperatures. From the merging histories we also compute statistical fluctuations of the  $L - T$  correlation.

*Subject headings:* galaxies: clusters – galaxies: intergalactic medium – hydrodynamics

## 1. Introduction

The X-ray emission from clusters of galaxies enables direct probing of gravitationally bound and virialized regions with virial radii  $R_v$  of a few Mpc, comprising total masses  $M \sim 10^{15} M_\odot$  mostly in dark matter (DM hereafter).

On the one hand, the X-ray temperature  $T \propto GM/R_v$  measures the depth of the potential wells. On the other hand, the bolometric luminosity  $L \propto n^2 R_X^3 T^{1/2}$  emitted as thermal bremsstrahlung by the intracluster plasma measures the baryon number density  $n$  within the volume  $R_X^3$ . The  $L - T$  relation constitutes a crucial *link* between the physics of the baryon component and the dynamical properties of the DM condensations.

The simplest model describing the former holds  $n$  to be proportional to the average DM density  $\rho$ , and  $R_X$  to  $R_v$ , so that  $n \propto \rho \propto M/R_v^3$  obtains (self-similar model, hereafter SS, Kaiser 1986). If so, the luminosity would scale as  $L \propto \rho^{1/2} T^2$ , which is inconsistent with the observed correlation close to  $L \propto T^3$  (Edge & Stewart 1991; Mushotzky 1994; Tsuru et al. 1996). Further steepening at the temperatures of galaxy groups is indicated for the emission not associated with single galaxies (Ponman et al. 1996). In addition, the SS model when combined with the standard hierarchical cosmogony (see Peebles 1993) yields for the clusters a local X-ray luminosity function too steep or too high compared with the data (Evrard & Henry 1991; Oukbir, Bartlett & Blanchard 1996).

So the indication is that the ratio  $n/\rho$  is to depend on  $M$  or  $T$ . It will be convenient to write the volume-averaged  $n^2$  in terms of two factors: the compression factor  $g(T) \geq 1$ , describing the gas overdensity relative to the outer value at the “boundary”, taken here to be at  $R_v$  as discussed in §3; and the inner shape factor  $I(R_v, T) \equiv R_v^{-3} \int_0^{R_v} d^3\mathbf{r} n^2(r, T)/n^2(R_v, T)$ , with the main contribution coming from inside

$R_X$ . The result is:

$$L \propto g^2(T) I(R_v, T) R_v^3 T^{1/2} \rho^2 \propto g^2(T) I(R_v, T) T^2 \rho^{1/2}, \quad (1)$$

where the last term follows from expressing  $R_v$  from  $T \propto M/R_v \propto \rho R_v^2$ . To obtain the *average*  $L - T$  relation to be compared with data, the factor  $g^2(T)$  in eq. (1) has to be averaged over the cluster histories, as we carry out in §2.3.

So, the difference of eq. (1) from the SS model has been factored out into the terms  $g^2(T)$  and  $I(R_v, T)$ , which are determined by the hydro- and thermodynamics of the gas in the forming cluster wells. At  $z \gtrsim 1$  the gas is expected to be *preheated* by feedback effects of star formation: injections of energy of stellar origin like Supernova winds eject the gas from the shallower potential wells, and heat the residual and the ejected gas to temperatures  $T_1 \lesssim 10^7$  K (Dekel & Silk 1986; Kaiser 1991; Ciotti et al. 1991; David et al. 1993, 1995; Cavaliere, Colafrancesco & Menci 1993). In addition, preheating is necessary to prevent too short cooling times (as pointed out by Cole 1991, and addressed by Blanchard, Valls Gabaud & Mamon 1992; see also White & Rees 1978) in early potential wells, shallower on average but containing denser gas. Subsequent evolution will lead to an increasing recovery of the universal baryonic fraction (White et al. 1993).

Previous attempts (Kaiser 1991; Evrard & Henry 1991) to tackle the  $L - T$  relation are based on the *extreme* assumption that the gas inside the X-ray core  $R_X$  is preheated but never subsequently shocked or mixed. Here we discuss the *other* extreme, i.e., the effects of shocks and mixing on the gas density inside clusters.

## 2. The Shock Model

As in the collapses the gas velocity becomes supersonic, shock fronts form at about  $R_v$ , and separate the infalling from the inner gas already at virial temperatures. In fact,

numerical simulations (see Evrard 1990; Takizawa & Mineshige 1997) of isotropic collapses show that, when the outer gas temperature is appreciably lower than the virial value  $T$ , a spherical shock front forms and, in the vicinity of  $R_v$ , slowly expands outwards leaving the gas nearly at rest and with a nearly flat temperature profile. When realistic, *anisotropic* collapses are considered (Navarro et al. 1996, Tormen 1996) shock fronts still form and convert into heat most of the hydrodynamical energy (Schindler & Müller 1993; Schindler & Böhringer 1993; Röttiger, Burnes, & Loken 1993).

Across the shock the gas entropy rises, and correspondingly a jump in the gas density and in the temperature is established from the exterior values  $n_1, T_1$  to the interior ones  $n_2, T_2$ . The density jump provides a *boundary* condition for the inner gas distribution in the form of the compression factor  $g(T_2/T_1) \equiv n_2/n_1$ . In addition, the interior temperature  $T_2$  governs, at equilibrium in a given gravitational potential, the inner density *profile* and hence the shape factor  $I(R_v, T)$ . These two effects enter eq. (1) for  $L$ , and will be discussed in turn.

## 2.1. The Compression Factor

The values of  $n_2, T_2$  and of the interior gas velocity  $v_2$  are related to their outer counterparts  $n_1, T_1$  and  $v_1$  by the requirements of mass, momentum and energy conservation across the shock. The plasma behaves as a perfect gas with three degrees of freedom, and the Hugoniot adiabat (see Landau & Lifshitz 1959) yields the compression factor

$$g\left(\frac{T_2}{T_1}\right) = 2\left(1 - \frac{T_1}{T_2}\right) + \left[4\left(1 - \frac{T_1}{T_2}\right)^2 + \frac{T_1}{T_2}\right]^{1/2}. \quad (2)$$

For strong shocks with  $T_2 \gg T_1$ , this saturates to the value  $g = 4$ , while for  $T_2 \rightarrow T_1$  it attains its lowest value  $g = 1$ .

The pre-shock temperature  $T_1$  is provided by the stellar (thermonuclear) energy

feedbacks recalled in §1, or by the virial (gravitational) temperature inside the clumps which are to merge with the cluster during its merging history considered below. In fact, the stellar feedbacks set for  $T_1$  the lower bound  $T_{1*}$  which we identify with the lowest temperatures (around 0.5 keV, Ponman et al. 1996) measured in groups.

Pre-shock temperatures of this order do not affect the rich clusters; instead, they affect the compression factor in the shallower potential wells with  $T \sim 1$  keV, as prevail at redshifts  $z \gtrsim 1$  but are also present at  $z \simeq 0$ . The full behavior of  $g(T_2/T_1)$  is shown by the dashed line in fig. 1 when  $T_1 = T_{1*}$  and the latter is in the range 0.5 – 0.8 keV. The actual values of  $T_1$  will be discussed in §2.3, taking into account the merging histories.

## 2.2. The Gas Disposition

The post-shock temperature  $T_2$  can be calculated from the pre-shock velocity  $v_1$ . The latter is driven by the gravitational potential  $V(r)$ , and reads  $v_1 = [-\alpha V(R_v)/m_p]^{1/2}$  with  $\alpha = 2[1 - V(R_m)/V(R_v)]$ . Here  $R_m$  is the radius where infall becomes nearly free; its upper bound is obtained by equating the Hubble flow to the free-fall velocity, which yields  $\alpha \approx 1.4$ . The post-shock condition is closely hydrostatic, i.e.,  $v_2 \ll v_1$ , as shown by the simulations. Then, using the equations in Landau & Lifshitz (1959) with the above value of  $v_1$ , we find

$$kT_2 \simeq -\frac{\alpha}{3} V(R_v) + \frac{7}{8} kT_1 . \quad (3)$$

The inner gas profile relative to that of DM,  $\rho(r)$  say, is governed at equilibrium by the scale-height ratio  $\beta \equiv \mu m_p \sigma_r^2 / kT_2$ , where  $\mu \approx 0.6$  is the gas mean molecular weight,  $m_p$  the proton mass, and  $\sigma_r$  the one-dimensional velocity dispersion of the DM. The profile

$$n(r) \propto [\rho(r)]^{\beta(T)} \quad (4)$$

(Cavaliere & Fusco-Femiano 1976), applies for a nearly flat  $T(r) \approx \text{const} \approx T_2$ .

The function  $\beta(T)$  entering the profile (4) is easily computed from eq. (3), for a given DM potential  $V(r)$  corresponding to  $\rho(r)$ . For the King potential (see Sarazin 1988)  $V(r) = -9 \mu m_p \sigma_r^2 r_c \ln[(r/r_c) + (1 + r^2/r_c^2)^{1/2}]/r$  with the core radius  $r_c = R_v/12$ , we obtain  $\beta(T)$  increasing somewhat from the value  $\beta \approx 0.5$  for  $T \approx T_1$  to  $\beta \approx 0.9$  for  $T \gg T_1$ . A similar result obtains using the potential proposed by Navarro et al. (1996). These two instances are illustrated in fig. 2, and demonstrate that in all cases the static gas density profile is *shallower* for lower  $T$  clusters.

### 2.3. $L - T$ from Merging Histories

For a given  $T_1$  the compression factor  $g$  is computed after eq. (2), and the shape factor  $I(R, T)$  is obtained by integration of  $n^2(r)$  computed after eq. (4); then the  $L - T$  relation may be obtained from eq. (1). But  $T_1$  depends on the thermal conditions of the infalling gas, which is preheated by Supernovae and *further* heated through virialization inside merging clumps.

In the former case, we take  $T_1 = T_{1*} = 0.5 - 0.8$  keV with a flat distribution, and obtain for  $g^2$  the dashed line in fig. 1. In the latter case, repeated merging events introduce fluctuations of  $T_1$  above  $T_{1*}$ , and hence of the interior density  $n_2$ , which modify  $g^2$ . The average effect is shown by the solid line in fig. 1, and the corresponding variance is illustrated by the shaded area.

To include both conditions, we take  $T_1$  to be the *higher* between the preheating value  $T_{1*}$  and the virial temperatures prevailing in the clumps accreted by the cluster. We perform a statistical convolution of the  $L - T$  relation over such merging histories, using Monte Carlo realizations of hierarchical merging trees of dark halos as introduced by Cole (1991). The code, written by one of us (P.T.), is based directly on the excursion set approach of

Bond et al. (1991) to the mass distribution.

For each merging event concerning a cluster with a current virial temperature  $T'$ , we compute  $g(T'/T_1)$  for the clumps being accreted, weighted with the associated mass fraction. We show in fig. 1 the quantity  $\langle g^2 \rangle$  averaged over the histories ending into a cluster of temperature  $T$ , along with its dispersion. These two quantities are used in eq. (1) to predict the average  $L - T$  correlation and its scatter. The results are shown in fig. 3, for the DM potential of Navarro et al. 1996; a different  $V(r)$ , like King's, only steepens somewhat the low- $T$  behavior.

### 3. Results and Discussion

Here we have proposed a model for the intracluster gas, to capture in a simple way one essential component of the complex gravitational systems constituted by groups and clusters. The model is focused on the formation of *shocks* between the the gas nearly in equilibrium with the cluster potential, and the infalling one. The latter is preheated by the release of thermonuclear energy, or by the gravitational energy in subclusters. Shock heating and compression determine the  $L - T$  relation.

The latter comprises both clusters and small groups in a single dependence, which smoothly *flattens* from  $L \propto T^5$  (for groups with  $T \lesssim 2$  keV), to  $L \propto T^{3.5}$  (for clusters with  $2 \text{ keV} \lesssim T \lesssim 7$  keV), toward  $L \propto T^2$  (for higher  $T$ ). Such behavior fits both the cluster data (Edge & Stewart 1991) and those for groups (Ponman et al. 1996) which – if considered separately – would require a much steeper  $L - T$  relation. Correspondingly, the volume-averaged baryonic fraction grows by a factor around 3 from small groups to rich clusters, but remains within 1.3 times the universal value.

In addition, we predict the gas density profiles to have a flat central region (the gas

“core”, see fig. 2), independently of the detailed shape of the DM potential; the profiles are actually *steeper* for larger virial temperatures. Correspondingly, the size  $R_X$  of the X-ray core (defined, e.g., at one half the integrated emission) grows with mass slower than  $M^{1/3}$ . The average cooling time within  $R_X$  exceeds the Hubble time out to  $z \approx 2$ , differently from the SS model.

We have checked that these results *persist* when we relax the approximation of a flat temperature profile in the cluster, and adopt instead a polytropic distribution with index  $\gamma$  ranging from 1 to 5/3; for  $\gamma > 1$  the temperature declines from the center toward the shock position.

In time, the shocked region expands and outgrows  $R_v$ , the infall velocity decreases, and the shock weakens with  $T_2$  approaching  $T_1$ . However, this occurs only over several dynamical times as shown by the N-body simulations (see Takizawa & Mineshige 1997); meanwhile, the shock positions remain close to  $R_v$ , as taken here.

We do not stress, instead, the  $z$ -dependence  $(1+z)^{1.5}$  of the normalization provided by the factor  $\rho^{1/2}(z)$  appearing in eq. (1). In fact, such dependence is easily swamped by the place-to-place variations of  $n_1$  and by the systematic increase in contrast of the large scale structures hosting groups and clusters (see Ramella, Geller, & Huchra 1992).

The robust predictions of the shock model do not require spherical symmetry, but only a small bulk velocity of the inner gas compared to  $(GM/m_p R_v)^{1/2}$ . Thus the model includes *anisotropic*, recurrent merging with other clumps of dark and baryonic matter.

The effects of extreme merging events are as follows. The few events involving *comparable* subclusters reshuffle the baryonic content and mix its entropy, but only moderately affect temperature and density. At the other extreme, the more isotropic accretion to a cluster of many *small* condensations *gravitationally* heated at temperatures

$T_1 \approx 1$  keV yields the highest compression and the largest contribution to the X-ray luminosity. Our *Monte Carlo* simulations span the range between these extremes.

The stellar preheating at  $T_{1*}$  of the external gas is essential to provide a lower limit for  $T_1$ . This ensures that the accreting gas, even when in a very shallow well or in a diffuse state, starts on a relatively high adiabat, corresponding to  $T_{1*} = 0.5 - 0.8$  keV. Such a heating of *thermonuclear* origin breaks down the otherwise self-similar form  $n/\rho = \text{const}$  of the ratio of gas to DM density to yield, at the boundary, the form  $n/\rho \propto g(T/T_1)$  shown in fig. 1. Values of  $T_{1*}$  smaller than 0.5 keV would lead, in a strictly self-similar evolution of DM halos, to  $L \propto T^2$  at variance with the data; larger values to a severe depletion of the gas and of the luminosities in groups and clusters.

The merging histories also produce the considerable scatter in the  $L - T$  relation shown in fig. 3, since the different virial temperatures of the stochastically merging clumps induce intrinsic variance in the internal density  $n_2$ . Further scatter may be contributed by the vagaries in the ambient density  $n_1$ , and by the possible lack of dynamical equilibrium in some groups, see discussion by Governato, Tozzi & Cavaliere 1996.

The shock model in the simple form presented here applies to the gas settled to equilibrium after each dynamical perturbation. This takes sound propagation times, somewhat shorter than the dynamical timescale taken anyway by the DM to adjust to equilibrium (Tormen 1996). The residual converging motions, even in spherical N-body simulations (Takizawa & Mineshige 1997), tend to balance the expansion of the shocked region to yield only small net velocities  $v_2 \approx 100$  km/s. These may be associated with some adiabatic compression of the central regions, but the resulting heating is only mild, as long as shocks form at radii of order  $R_v$ .

The other extreme is tackled by the model proposed by Kaiser (1991) and refined by Evrard & Henry (1991). This assumes that, after preheating, the central cluster region

visible in X-rays contains the same gas (about 10 % of the present total) engaged in a smooth adiabatic compression. However, we find only 5% of the largest single progenitors to have masses (DM and hence gas) exceeding 10% of the present values at  $z \geq 1.5$ , when most stellar preheating takes place. On the other hand, N-body simulations (see refs. in §2) and observations (see Zabludoff & Zaritsky 1995) show that each cluster history includes a few merging events between comparable structures, which will reset the core gas to a higher adiabat.

The adiabatic model (with preheating) predicts a single, scale-free relation  $L \propto T^{3.5}$ , or  $L \propto T^3$  if the gas equilibrium holds out to  $R_v$  as in Evrard & Henry (1991). However, on the largest scales that ought to sample fairly the universal baryonic fraction (White et al. 1993) one expects saturation toward the scaling  $L \propto T^2$  of the SS model; in addition , at the group scales a much steeper dependence is indicated by observations. Such opposite departures of the  $L - T$  correlation from a single power-law are beyond the reach of the adiabatic model, but within the predictions of the shock model.

The model we propose leads (Cavaliere, Menci, Tozzi 1997) to a local luminosity function  $N(L, z = 0)$  in agreement with the data, and to  $N(L, z)$  with the mild or no evolution shown by recent data, and confirmed by the deep X-ray counts.

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Fig. 1.— The square compression factor  $g^2(T/T_1)$  is shown by the dashed line when  $T_1$  takes on its lowest value  $T_{1*}$ , with the latter uniformly distributed in the range  $0.5 - 0.8$  keV. The different values of  $T_1$  due to the merging histories (discussed in §2.3) affect the average dependence as shown by the solid line, and provide the  $2\sigma$  dispersion shown by the shaded region. A tilted CDM power spectrum in a critical cosmology (see White et al. 1996) has been used.

Fig. 2.— The gas density profile (for a uniform  $T$ ) derived from eqs. (3) and (4) using the King DM potential (upper panel), or (lower panel) that given by Navarro et al. (1996). The dotted lines refer to a group with  $T = T_1 = 0.8$  keV, and the solid lines to a rich cluster with  $T_1 \ll T = 8$  keV.

Fig. 3.— The L-T relation from the shock model, convolved with the merging histories of DM halos, is compared with data for clusters of galaxies (filled squares, from Edge & Stewart 1991), and for groups (open squares, from Ponman et al. 1996). The DM potential of Navarro et al. (1996) is used. The shaded region outlines the  $2 - \sigma$  scatter expected from the merging histories.





